and common-sense conclusions, who have an equipment of technical knowledge and who can produce results.

In discussing the teaching of mathematics to engineers, we should emphasize not the mathematics nor the engineers, but the teaching. Aside from the imparting of knowledge and technical ability, the teaching of mathematics gives opportunity for training in the use of logical methods and in the drawing of intelligent conclusions from unorganized data which will make efficient men, whether they follow pure engineering, or semi-technical, or business pursuits. Such teaching does not come from the text-book; it must be personal—it comes from the teacher. He must be in sympathy with engineering work and have a just appreciation of its problems and its methods. He must be imbued with the spirit and the ideals of the engineer.

Chas. F. Scott

THE POINT OF VIEW IN TEACHING ENGINEERING MATHEMATICS[1]

I hardly know why I should have been asked to address you at this conference. Possibly, however, the fact that I am a civil engineer by profession, without having been permitted ever to practise this profession, and the additional fact that I have been a professional teacher of mathematical physics, without having been permitted to continue in this work, have led your committee to think that I might furnish a conspicuous illustration of the failures to which colleges and universities may lead in these lines of endeavor.

Having listened attentively to the three formal papers just read, I find it essential to revise my program and instead of following similar lines to those of the preceding speakers, it seems essential to take direct issue with them. This I am disposed to do, not so much because I differ wholly from the views they have set forth, as because it seems necessary to have other sides of the questions they have discussed represented. The preceding speakers appear to me to have taken themselves somewhat too seriously. This is a general fault of both theoretical and practical educationalists. My own experience leads me to conclude that in educational affairs the teacher, the school, the college and the university play a much less important rôle than we commonly suppose. In fact, I have reached the provisional conclusion that the majority of our students turn out fairly well in the world not so much by reason of the academic instruction they receive as in spite of it.

My impression also is that in taking ourselves too seriously as teachers of one subject or another, we have, as a rule, quite underestimated the magnitude and the difficulty of the psychological problems with which we have to deal. We have, as a rule, quite overestimated the capacity of our average student, and have thus usually expected too much from him. It is, of course, desirable to set our ideal high and try to rise to an elevated intellectual level; but in doing so we have commonly neglected the influence of heredity as well as of environment. I am inclined to think Dr. Holmes was right when he said that it is essential in the generation of a gentleman to begin four hundred years before he is born. So also is it necessary, if we wish to develop a student into a first-class scholar, to begin back some generations before we take up the formal work of training in our colleges or schools of engineering. It is an important fact, also too commonly over-

[1] Extempore remarks before Sections A and D of the American Association for the Advancement of Science and the Chicago Section of the American Mathematical Society, at the Chicago meeting, December 30, 1907.

looked, that the fundamental ideas involved in the mathematics and in the mathematical physics essential to the preliminary training of a prospective engineer are far more difficult of comprehension than we are wont to suppose. As a rule, I think we begin our elementary mathematics somewhat too early for the average mind. The result is that our students acquire a mere literary knowledge of the subject without grasping the basic ideas essential to clear thought and especially essential to applications. I am going to give you some illustrations of this fact. They will show how difficult it is for the average mind to attain a proper understanding of mathematico-physical concepts. The difficulties here are much the same as the difficulties of grammar. As you know, children learn to speak, and often speak very well, long before they know anything of formal grammar, and this is the natural mode of development, for the logic and subtleties of grammar can be appreciated only by rather mature minds.

But if the concepts which belong to the study of language and of grammar are rather formidable, those which belong to the higher mathematics and mathematical physics are profoundly more difficult of adequate comprehension. Let me illustrate this point by a citation from experience furnished by the case of a graduate from one of our universities who presented himself to me a few years ago, while I was dean of a graduate school of Columbia University, as a candidate for a higher degree in mathematical physics. This student had studied mechanics and had attained a degree in engineering. In order to learn something of the breadth and depth of his knowledge, I asked him what it is that makes the trolley car run after the current is cut off. He answered, "It is the force of the momentum of the power of the energy of the car." There is no reason to suppose that he had not received good mathematical and physical training, and yet it is plain from the answer he gave me that he knew next to nothing of the meaning of the terms he used. I may cite another case of a successful practising engineer, who was a pupil of no less authorities in mechanics and engineering than Lord Kelvin and Rankine. This man wrote me a letter in which he sought to convince me that Newton and his followers are all wrong with regard to the parallelogram of impulses. "Thus," he said in his letter, "if a particle start out from a given point under the simultaneous action of two impulses, it will not move in the parallelogram of the impulses, but it will move in a tautochronous, brachistochronic, plane catenary curve of a resilient character."

These illustrations show how extremely difficult it is to master the fundamental ideas which belong to a great science; and the difficulties are so great that I am disposed to excuse, or at any rate palliate, the blunders made by our average student. He is, in fact, with all his blunders, not very far behind many of his teachers, for it is not uncommon for them to use in their lectures and text-books words not at all free from ambiguity. Witness, in fact, the loose use of such words as force, power, pressure, stress, and strain in some of the best text-books and treatises of the nineteenth century. The word "power," for example, is often used in two radically different senses in the same sentence.

These difficulties and ambiguities lead me to suggest, in opposition to the precepts laid down by a previous speaker, that we may well consider the desirability of printing mathematical books free from demonstrations but containing plain statements of facts. I have used such books myself and am disposed to think they are amongst

the best books we may place in the hands of a student. The simple fact is that we do not follow a logical order of development in acquiring knowledge. We proceed rather by the method of "trial and error," and we often find out the facts with regard to an item of learning long before we become aware of the principle involved.

Hence I think the reason why few of our engineers know much about the formalities of mathematics and mathematical physics after they get through college is plain enough. They are driven over so many subjects during the four years of their college life that they have little or no time for reflection. This latter must come later in life when the mind has developed a sufficient degree of maturity to appreciate the more recondite principles which lie at the foundation of all the higher learning. This fact is well illustrated also by the case of our friends, the humanists, who have, as you know, for a long time proposed the study of geometry for "mental discipline." As a matter of fact, those who have acquired anything like a grasp of geometrical principles know that very few students of Euclidean geometry acquire anything like an adequate appreciation of the ideas involved, and it is only in the rarest instances that these students pursue the subject after leaving college.

I have not much sympathy with the engineers who would like to have their own kind of mathematics, and I am not disposed to commend very highly the works on calculus and other branches of pure mathematics designed especially for engineers. On the other hand, our modern mathematicians have generally failed to understand the needs of the engineer. Our more recent type of mathematician has devoted himself too largely to the refined questions of convergence and divergence of series and of existence theorems to properly equip him for the numerous and important applications which the ideal engineer should be able to make of his mathematical knowledge. The modern mathematician seems prone to make the engineer with some degree of mathematical talent afraid of himself. I have met some students whose early training had filled them with caution to such a degree that they would not use infinite series for fear that a divergent one might be encountered. It is known, however, as a matter of fact, that most series essential in the applications of mathematics to mathematical physics are safe in this regard, and one of the best ways for the elementary student to learn of the degree of convergence is to apply numerical computation to these series.

This leads me to say a few words concerning numerical computations, in which very few engineers and still fewer mathematicians show any degree of proficiency. It seems to me this is one of the most lamentable defects of our elementary teaching in mathematics, though here as elsewhere the intrinsic difficulties are much greater than we commonly suppose. This fact is in evidence at almost every meeting of our scientific societies, for it oftenest happens that the author of a paper involving numerical calculation will talk of the decimals involved instead of the significant figures. Thus, he will say, "this result is correct to five places of decimals," when he should say, "this result is correct to a specified number of significant figures," the latter form of expression being requisite to indicate the degree of precision attained. There is a grave defect in our elementary teaching in these matters; but it arises from the fact that almost none of our teachers of elementary mathematics are qualified to understand the refinements and the difficulties of precision in compu-

tation. Thus, it often happens that students will give results to five or seven significant figures when the data do not justify any such apparent precision.

To correct these evils we must have a convention of mathematicians, engineers and professional computers who will show authors how to produce elementary text-books giving adequate attention to these matters.

As regards numerical computation, there is in general need of more practise, since it is through the concrete that we learn of the abstract and the fundamental. No important formula in any text-book or treatise should go without an appropriate illustrative numerical example.

I would like to take advantage of this occasion to express a hope with regard to the future of our country and to the possibility of development which may come through suitable cooperation between mathematicians and engineers. Nothing delights me more than to attend a meeting of this kind where mathematicians and engineers have come together. It is an auspicious sign of the times. It is one of the results I have been looking forward to for the past thirty or forty years. Some of us here are old enough to have lived in two epochs, namely, the pre-scientific and the present epoch. We can remember a time when engineers could not have got a hearing such as they have to-day. The history of their rise and development, at least in this country, is well known to some of us. It dates back to a time only about forty years ago. During this time the engineers have fought their way forward to the position now accorded them in contemporary society. They have won a place in public esteem without which it would have been impossible to hold such a conference as we are holding to-day. This esteem has been won in spite of much opposition, coming especially from the older academic institutions; but now having attained adequate recognition especially as practising engineers, we have a much higher duty to perform, and this I trust we shall be able to meet adequately through cooperation with our friends the pure mathematicians. I know of no work more important to the general advancement of mathematico-physical science than that which may lead to the development of mathematical physicists, men who possess at once good mathematical knowledge and correspondingly adequate equipment in physical science. Here is a field greatly in need of concentrated effort and of adequate appreciation. It is a lamentable fact that while we can easily develop pure mathematicians of a high order and experimental physicists of an equally high order, it seems very difficult for us to develop minds possessing both qualities. To a large extent I think the development of pure mathematics in the future will depend, as in the past, on the stimulus furnished by mathematico-physical ideas; and in like manner success in the development of mathematical physics will depend equally in the future on mathematical ability of the highest order. In this line of work we Americans have not done our full duty, and it behooves us as mathematicians and engineers, now that we have got together on the plane of mutual interest, to give attention to this important field of work.

The French engineers led by Navier and followed by Lamé, Clapyron, and especially by the "dean of elasticians," Barré de Saint-Venant, have contributed to science the most important branch of mathematical physics, namely, what is commonly called the theory of elasticity. This is superbly difficult in its purely mathematical aspects and exquisitely beautiful in its physical aspects, and it stands as a splendid example of the possibilities which

may result from adequate cooperation between mathematicians and engineers.

The chief difficulty in the way of developing mathematical physicists appears to lie in the inadequate appreciation of this type of work by contemporary society. Pure mathematics has a prestige of more than twenty centuries behind it, and the practical work of the engineer appeals even to the dullest of intellects; but we have failed thus far, in this country especially, to adequately esteem the worker in the intermediate field. We must look to it that more attention is given to this field in our colleges and universities. Every university should have two or three men eminent in mathematical physics as well as two or three men eminent in pure mathematics. Thus, while I would not advocate the pursuit of pure mathematics or the pursuit of practical engineering less, I would urge the pursuit of mathematical physics more. It is only by the cultivation of this branch of study and investigation that we can keep alive the sources of engineering knowledge. Important and indispensable as the practical work of the engineer is, the cultivation of investigation and discovery in his science is still more important and indispensable. Hence I would urge that when the more pressing questions of elementary instruction in mathematics and engineering have been adjusted, we give attention to the more inspiring and more important questions of the clarification and enlargement of the fundamental ideas of our sciences.

R. S. Woodward

THE CHICAGO ACADEMY OF SCIENCES[1]

In his historical sketch of the academy, published in 1902, Mr. W. K. Higley, late secretary of the academy, divides the history of the institution into three periods, first, that preceding the fire of 1871; second, that between the time of the fire and the erection of the present building in Lincoln Park, and third, the period dating from the occupancy of these new quarters. It is often of interest and value to cast a retrospective glance over a period of years in the history of an institution, in order that a clear idea may be obtained of the value of the operations. It is now thirteen and a half years since the building was first occupied and the intervening years have witnessed a steady growth in the collections and also in the interest of the academy's work among the citizens of the community and of the city. The collections which were turned over to the curator in July, 1894, were comparatively small in number, although containing some very interesting and valuable material. The records show an aggregation of about 55,000 specimens on this date. In the thirteen and a half years this number has been increased fourfold; the number of specimens in the museum January 1, 1908, being 226,781, or an increase of 171,781 specimens. It must be remembered that the majority of these additions were presented by the owners or collectors, as there has been no fund for the purchase of specimens. It must not be forgotten, however, that several members of the academy, like our deceased patron, Mr. George H. Laflin, have from time to time given money for the purchase of material, but the entire amount spent for such purposes has not exceeded $5,000. A part of the collections have been secured by the museum staff while on their vacations.

An analysis of the additions shows that three departments head the list in the number of additions:

Mollusca	89,757
Insects	33,914
Paleontology	21,145

The first named contains several types, a larger number of cotypes and a very extensive series of autotypes and locotypes, as well as many rare species and development series. The same is true of the fossil collection and to a less degree of the insect collection. The

[1] Extracted from the annual report of the secretary.